$$\int_{1,1}^{e} \frac{1}{x\ln(x)}\,dx = \int_{1,1}^{e} \frac{1}{x} \times \frac{1}{\ln(x)}\,dx$$

$$\frac{u'}{u} \xrightarrow[P]{} \ln(|u|)$$

$$= \int_{1,1}^{e} \frac{\frac{1}{x}}{\ln(x)}\,dx$$

$$= \Big[\ln(\ln(x))\Big]_{1,1}^{e}$$

$$= \ln(\ln(e)) - \ln(\ln(1,1))$$

$$= -\ln(\ln(1,1)).$$

$$\int_5^6 \frac{2x}{x^2+1}\,dx = \Big[\ln(x^2+1)\Big]_5^6$$

$$= \ln(37) - \ln(26)$$

$$= \ln\left(\frac{37}{26}\right).$$

$$\frac{u'}{u}$$

$$\int_0^1 \frac{3x}{2x^2+3}\,dx = \int_0^1 \frac{3}{4} \times \frac{4x}{2x^2+3}\,dx$$

$u(x) = 2x^2+3$

$u'(x) = 4x$

$$= \frac{3}{4}\int_0^1 \frac{4x}{2x^2+3}\,dx$$

$$= \frac{3}{4}\Big[\ln(2x^2+3)\Big]_0^1$$

$$= \frac{3}{4}\big(\ln(5) - \ln(3)\big)$$

$$= \frac{3}{4}\ln\left(\frac{5}{3}\right).$$

$$\int_0^1 \frac{5x}{\sqrt{3x^2+1}}\,dx = \int_0^1 \frac{5\times 2}{6} \times \frac{6x}{2\sqrt{3x^2+1}}\,dx \qquad (\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$u(x) = 3x^2+1$

$u'(x) = 6x$

$$= \frac{5}{3}\int_0^1 \frac{6x}{2\sqrt{3x^2+1}}\,dx$$

$$= \frac{5}{3}\left[\sqrt{3x^2+1}\right]_0^1$$

$$= \frac{5}{3}(2-1)$$

$$= \frac{5}{3}.$$

$$\int_{-1}^1 \sin(3x)\cos^2(3x)\,dx = -\frac{1}{3}\int_{-1}^1 -3\sin(3x)\cos^2(3x)\,dx$$

$u(x) = \cos(3x)$

$u'(x) = -3\sin(3x)$

$u'u^2 \xrightarrow[P]{} \frac{1}{3}u^3$

$$= -\frac{1}{3}\left[\frac{1}{3}\cos^3(3x)\right]_{-1}^1$$

$$= -\frac{1}{9}\left(\cos^3(3) - \cos^3(-3)\right)$$

$$= 0.$$

$$\int_0^1 x e^x \, dx = \left[x e^x\right]_0^1 - \int_0^1 e^x \, dx = e - \left[e^x\right]_0^1 = e - (e-1) = 1 .$$

$$\left| \begin{array}{ll} u(x) = x & u'(x) = 1 \\ v'(x) = e^x & V(x) = e^x \end{array} \right.$$

$$\int_a^b uv' = \left[uv\right]_a^b - \int_a^b u'v$$

$$\int_0^x t e^t \, dt = \left[t e^t\right]_0^x - \int_0^x e^t \, dt = x e^x - \left[e^t\right]_0^x = x e^x - (e^x - 1)$$

$$= x e^x - e^x + 1$$

$$= (x-1) e^x + 1$$

$$g(x) = (x-1) e^x + 1$$

$$g'(x) = 1 \times e^x + (x-1) e^x + 0 = x e^x .$$

$$\int_0^{\pi} x \cos(x) \, dx = \left[x \sin(x)\right]_0^{\pi} - \int_0^{\pi} \sin(x) \, dx$$

$$\left| \begin{array}{ll} u = x & u' = 1 \\ v' = \cos(x) & V = \sin(x) \end{array} \right.$$

$$= 0 + \left[+\cos(x)\right]_0^{\pi}$$

$$= \cos(\pi) - \cos(0)$$

$$= -2 .$$

$$\int_1^x \ln(t) \, dt = \int_1^x 1 \times \ln(t) \, dt = \left[t \ln(t)\right]_1^x - \int_1^x 1 \times dt$$

$$\left| \begin{array}{ll} u = \ln(t) & u' = \frac{1}{t} \\ v' = 1 & V = t \end{array} \right.$$

$$= x \ln(x) - \left[t\right]_1^x$$

$$= \underbrace{x \ln(x) - x}_{} + 1$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos(x)\,dx = \left[x^2 \sin(x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \sin(x)\,dx$$

$$\left| \begin{array}{ll} u = x^2 & u' = 2x \\ v' = \cos(x) & v = \sin(x) \end{array} \right.$$

$$= \frac{\pi^2}{2} - 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \sin(x)\,dx$$

$$\left| \begin{array}{ll} u = x & u' = 1 \\ v' = \sin(x) & v = -\cos(x) \end{array} \right.$$

$$= \frac{\pi^2}{2} - 2\left(\left[-x\cos(x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)\,dx\right)$$

$$= \frac{\pi^2}{2} - 2\left(0 + \left[\sin(x)\right]_{-\frac{\pi}{2}}^{\pi/2}\right)$$

$$= \frac{\pi^2}{2} - 2 \times 2$$

$$= \frac{\pi^2}{2} - 4.$$